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The Statistics of Natural Shapes in Modern Coral Reef Landscapes

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ABSTRACT
Spatial heterogeneity is a fundamental characteristic of modern and ancient depositional settings, and the scaling of many carbonate environments has been shown to follow power function distributions. The difficulty in obtaining information on the horizontal persistence of sedimentary lithotopes at the basin scale has, however, hampered evaluation of this fact over larger geographic areas. In recent years, large-scale maps of reef facies derived from remotely sensed data have become widely available, allowing for an analysis of reef-scale map products from 26 sites spread through four reef provinces, covering >7000 km$^2$ of shallow-water habitat in the U.S. territorial Pacific. For each site, facies maps were decomposed to polygons describing the perimeter of patches of differing sedimentologic/benthic character. A suite of geospatial metrics quantifying unit shape, fractal dimension, and frequency-area relations was applied to investigate the intra- and intersite variability. The spatial architecture of these reef sites displays robust fractal properties over an extended range of scales with remarkable consistency between provinces. These results indicate the possibility of extrapolating information from large to small scales in various depositional environments.

Introduction
The existence of scaling laws in Earth systems is well documented (Turcotte 1989; Van Gardingen et al. 1997; Kunin 1998; Wu 1999; Hergarten 2002; among many others). Following the definition of Urban (2005), we define “scaling” to mean the explicit extrapolation of details at large extent and coarse scale to their implications at small extent and generally finer scale (or vice versa). Under the premise that aspects of the structure of reef landscapes adhere to scaling laws (e.g., Wilkinson and Drummond 2004; Purkis et al. 2005), there exists the possibility to extrapolate observations made at the large [observable] scale to predict landscape properties at the fine [nonobservable] scale. In essence, this facilitates prediction of the behavior of a system beyond the scale of observation and possibly at a resolution that is difficult or impossible to examine using conventional methodology. This aspect of depositional settings is highly relevant considering that our present arsenal of tools capable of resolving the anatomy of sedimentary environments is inherently scale limited (space-borne and airborne remote sensing in exposed settings, reflection seismics in the subsurface). However, it is these elusive fine-scale processes that largely govern the functionality of almost any sedimentary landscape in modern systems and therefore dictate the architecture and porosity of buried sedimentary successions. The latter observation is pertinent considering the fact that sediments host virtually all of the world’s reserves of hydrocarbons and comprise the largest aquifers on Earth.

For geologists, data on the horizontal persistence of various sedimentary facies are difficult to obtain from outcrops and in the subsurface (e.g., Wilkinson et al. 1999). However, the mechanisms of deposition in modern environments are usually assumed to be essentially the same as for buried ancient systems. The spatial patterns in modern analogues can therefore assist interpretation of the inaccessible ancient (e.g., Harris 1996). However, by virtue of the expansive and remote nature of...
many modern depositional settings, collecting fine-scale information that is representative of system-scale processes is impractical. Therefore, a model capable of predicting fine-scale processes from system-scale observations would be of great value. For carbonate (as well as clastic) landscapes, satellite and airborne remote sensing offers an efficient way to quantify the spatial complexity of depositional systems in shallow waters (<30 m) at regional extent in plan view (Harris and Kowalik 1994; Wilkinson and Drummond 2004; Purkis 2005; Lidz et al. 2006) and is therefore a suitable data source from which to derive such models. In recent years, a comprehensive database of modern reef settings has been constructed through analysis of high-resolution optical satellite imagery. The main conduit for this work has been the biogeography initiatives of the National Oceanic and Atmospheric Administration (NOAA), which supplies map products for the reefs of the U.S. territories as geographic information system–ready categorical vector data (polygon maps). Satellite imagery depicting the location and distribution of shallow-water coral ecosystems is analyzed in conjunction with multibeam and airborne lidar bathymetry data sets to apportion benthic characters into dominant facies classes.

Here we present a basin-scale analysis of shape, size, and geometric complexity of benthic habitats for Pacific reef sites covering >7000 km². For each site, the benthic landscape is decomposed into individual polygons describing the perimeter of patches of differing substrate character. Each polygon is subsequently processed to extract statistics of area, geometric shape, and perimeter complexity. Approximately 6500 polygons were considered, comprising eight facies assemblages.

The resulting database is used to investigate three main questions: [1] Is patchiness in Pacific reefs predictable by virtue of the existence of scaling laws in habitat spatial configuration? [2] Which aspects of the architecture of carbonate lithotopes conform to a fractal? [3] What are the likely mechanisms promoting the observed behavior?

**Study Areas**

Four Pacific reef provinces are considered in this analysis: [1] Guam, [2] the chain of 14 islands comprising the Commonwealth of the Northern Mariana Islands (CNMI), [3] the four islands of American Samoa, and [4] the seven islands of the Hawaiian chain (figs. 1, 2). NOAA benthic map products (e.g., fig. 3) are independently assessed for their thematic accuracy and have an accuracy of 85%–95% for habitat cover mapped with a minimum mapping unit of 1 acre. The validity of this accuracy was verified by S. J. Purkis and B. M. Riegl at multiple sites during a 2004 field campaign to Saipan, Tinian, Agrihan, and Rota in the CNMI. All resulting map products were rendered in GIS shapefile format, and the described analysis was conducted using MATLAB software.

**Methods**

**Preprocessing.** Before we quantified patchiness, the shapefile database was processed to remove any artifacts that would compromise the analysis. The NOAA map products, for example, frequently contained multiple internal boundaries within a polygon that represented a single facies class. Internal boundaries and polygons delineating emergent features were removed, ensuring that each polygon represented a single seabed class of uniform sediment type. Second, the typology of the landscape was verified, ensuring that internal polygons represented holes within parent polygons and thus would be treated as “negative” space during subsequent analysis.

**Patch Analysis.** Patch analysis can be used to describe many spatial aspects of landscapes that are intuitively apparent but difficult to quantify. Landscape metrics are highly sensitive to scale; that is, assessment of the structure of the landscape may change with the grain (resolution) and extent (area covered) of the map on which they are calculated (Wagner and Fortin 2005). However, if a degree of consistency across scale is observed, the landscape can be described as scale invariant, or fractal. Patch analysis calculates statistics for each polygon in the map, and patch-level descriptors were used to quantify the size (area), shape, and complexity of the reef systems. The resulting data were used to investigate spatial patterns of the shape statistics and to provide the basic measurements for landscape analysis.

**Scale Independence in Pacific Reef Provinces**

**Patch Frequency–Area Relations.** As is the case with many segmented terrains, the analyzed reef landscapes are characterized by a large number of patches of small size and a small number of large size. Plotted in the log-log domain (fig. 4), frequency of all facies decreases with increasing patch area as a power law:

$$\log(f) = \log(A)^\alpha + \beta,$$  \hspace{1cm} (1)
Figure 1. Locations of the 26 studied sites in the reef provinces of Guam and the Northern Marianas Islands, American Samoa, and the Hawaiian archipelago.
**Figure 2.** Overview of the 26 studied reefscapes within the four Pacific provinces: [1] Commonwealth of the Northern Marianas Islands, [2] Guam, [3] American Samoa, and [4] Hawaii. Total reef area for all sites exceeds 7000 km$^2$. Note that each scale bar is different.
Figure 3. Subsection of the National Oceanic and Atmospheric Administration benthic habitat map of the fringing reefs of Tutuila (American Samoa) showing the spatial configuration of the five dominant facies types in the area. In a natural landscape such as this, variation is often continuous and exists between patches that rarely have well-defined edges. However, the simplification of the landscape into a patch mosaic is a good and appropriate method to assess facies distributions.
Figure 4. A, Relations between patch frequency and area by lithotope type for all Pacific sites. Bin size is 500 m², and total number of patches is 6439. Slopes of the regressions are facies specific but lie within a relatively narrow range (table 1). The minimum size of all lithotope units is partially constrained by the 1-acre (~4050 m²) minimum mapping unit (MMU) used to create the National Oceanic and Atmospheric Administration data set, with the majority of patches attaining minimum sizes twice as large as the MMU. Although maximum patch sizes of 10⁸ m² are attained by some facies in the data set, the analysis is truncated at 10⁶ m² because larger patches are too rare to be representative of the pattern at lower orders. B, Slope of the regression line describes habitat patchiness. Despite differing area-frequency relations between facies, the relative degree of patchiness is remarkably consistent.

where \( f \) is the number of patches having area \( A \), \( m \) is the slope of the regression line \( (<0) \), and \( b \) is the \( Y \)-intercept. The fractal dimension \( D \) of the frequency-area distribution is defined as \( 1 - m \). The relationship is thus indicative of a feature with a constant \( D \) over a wide range of patch areas, with the implication being that not only do patches become more abundant with decreasing area but also they do so at a predictable rate. Exponential and logarithmic distributions are both proportional (rather than constant) changes in \( x \) or \( y \); exponential distributions describe some proportional change in \( y \) relative to a constant change in \( x \), and logarithmic distributions describe some proportional change in \( x \) relative to a constant change in \( y \). Power functions are distributions that describe proportional (rather than constant) changes in \( x \) and \( y \); they describe some proportional change in \( y \) relative to some proportional change in \( x \). When polygons are considered according to the facies classes that they describe, it is clear that both solid substrates (corals and hardgrounds) and unconsolidated sand (with algal and seagrass communities) display a power law relationship between patch frequency and area.

Slopes and intercepts of the regression lines are distinctly facies specific (fig. 4). Seagrass meadows are conspicuous by attaining a maximum patch size of only \( 10^5 \) m², an order of magnitude less areal coverage than that of the other substrates. Consequently, the regression line characterizing seagrass is noticeably offset from those describing other habitats. A shallow slope of the regression line would indicate a regime including a wide range of area values, each with comparable abundance. In contrast, a steep slope denotes a system whose patches are of similar size with disparate frequencies (fig. 4B). The slopes of the regression lines are similar for the different substrates, indicating a like degree of patchiness (table 1), although the total areal coverage differs by facies.

The high \( r^2 \) value of each regression indicates a similar degree of patchiness for any given facies type. This is perhaps surprising considering that the data set is built on habitat patchiness extracted from potentially dissimilar depositional settings spanning 60° of longitude and ranging in extent from the narrow and relatively impoverished fringing reefs of the northern limits of the CNMI to the expansive and diverse systems of the Hawaiian archipelago and Samoa. The implication is that the degree of within-site variance in landscape patchiness appears independent of the extent of the host reef system or its geographical position. Therefore, if incomplete data sets do not show the full extent of a depositional system (as is common in the sub-surface), patch frequency relationships cannot reveal whether the visible mosaic is just a small part of a larger landscape, or the entire system.
Patterns in Complexity and Shape. When considering mosaics of facies that make up the gross geomorphologies of the reefal provinces, the shape of individual lithotope units is less predictable than area-frequency relations. At the small scale, patch shape is dictated by a stochastic and multiscale interplay of biotic growth, larval dispersal, and settlement transport across hydrodynamic gradients. Such multiscale randomness is, however, known to seed landscapes that fit power law distributions for patchiness and complexity [Pascual et al. 2002; Urban 2005], properties consistent with a fractal [Li 2000]. As the size of a lithotope unit approaches the dimension of the shelf on which it sits, by necessity it will adopt the (typically elongate) form of this host shelf. Shape may therefore be predictable for large lithotope patches. Additionally, there are no obvious reasons why the degree of boundary complexity should be skewed to systematically high or low values for small or large patches.

The Abundance of Intricacy. We used the box-counting method [as detailed in Turcotte 1997; Purkis et al. 2005; Schlager 2005] to quantify polygon complexity in the Pacific reef data set. This metric measures the tortuosity [wiggliness] of a boundary between adjacent lithotopes. The calculation is performed by iteratively covering the boundary with progressively smaller boxes of side length \( \delta \) and tallying the number \( N \) intersected by the curve boundary at each iteration [fig. 5]. If \( N \) increases proportionally to the reduction in \( \delta \), the relation is described by a power function, and it is inferred that the boundary is a fractal.

The box-counted fractal dimension of the boundary \( D_s \) is taken as the slope of the regression line (in a log-log plot) relating \( N / \delta \) to \( 1 / \delta \) [fig. 5]. The number of box-reduction cycles over which \( \delta \) decreases in proportion to increasing \( N / \delta \) is termed the “fractal span” \( D_s \) of the boundary. The duration of this proportional relationship is determined using the correlation coefficient \( r^2 \) between the two variables. An \( r^2 \) value of 0.99 was set as the tolerance below which the relation is not considered to adhere to a power function. The \( D_s \) of the object is therefore the number of iterations completed with \( r^2 \geq 0.99 \). We use both \( D_s \) and \( D_B \) as proxies for lithotope complexity, with greater values denoting increasing complexity; \( D_s \) is most useful in cases where a continuous measure of complexity is required [e.g., exploring complexity-shape relations], as opposed to the integer values of \( D_B \) which can be used to appraise groups of objects of similar complexity [i.e., families of patches that have the same \( D_s \) value].

The complexity [as quantified by \( D_s \)] of patch boundaries is shown to be scale independent [fig. 6]. Polygons of high complexity [more intricate boundaries] have a greater fractal span \( D_s \) than simple, noncomplex ones. Graphing \( D_s \) versus the area of all polygons having that \( D_s \) value [fig. 6] indicates that there is no pronounced change in complexity with increasing area; more complex polygons do not have a significant likelihood of being larger or smaller. Complexity and scale are decoupled and fit the model of a fractal system.

Although patch complexity is largely independent of patch area, the probability of encountering a simple or complex patch in the considered landscapes is not equal. All facies types follow a first-order trend of being composed largely of patches of simple geometric shape [i.e., a high probability that \( D_s \) is low in fig. 7]. Highly intricate patch geometries [high \( D_s \)] are, however, present at a similar and low-level degree for all facies types. In all cases, the relation between increasing patch complexity

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**Table 1.** Regression Metrics for the Patch Frequency–Area Relations Given in Figure 4

<table>
<thead>
<tr>
<th>Lithotope</th>
<th>Slope</th>
<th>Intercept</th>
<th>( r^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uncolonized hardground</td>
<td>-2.54</td>
<td>37.75</td>
<td>.98</td>
</tr>
<tr>
<td>Macroalgae</td>
<td>-2.09</td>
<td>30.72</td>
<td>.95</td>
</tr>
<tr>
<td>Sand</td>
<td>-1.78</td>
<td>26.44</td>
<td>.95</td>
</tr>
<tr>
<td>Coral</td>
<td>-1.58</td>
<td>23.29</td>
<td>.96</td>
</tr>
<tr>
<td>Turfing algae</td>
<td>-1.73</td>
<td>24.13</td>
<td>.93</td>
</tr>
<tr>
<td>Colonized hardbottom</td>
<td>-1.45</td>
<td>20.91</td>
<td>.94</td>
</tr>
<tr>
<td>Coralline algae</td>
<td>-1.46</td>
<td>19.94</td>
<td>.95</td>
</tr>
<tr>
<td>Seagrass</td>
<td>-0.98</td>
<td>12.38</td>
<td>.93</td>
</tr>
</tbody>
</table>

Note. The fractal dimension \( D_s \) of the frequency-area distribution is defined as \( 1 - \text{slope} \). Intercepts denote the log of the frequency when \( \log(\text{area}) = 0 \). The \( r^2 \) values exceed 0.93 in all cases, indicating a strong fit to the power law.

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Figure 5. Estimating the fractal dimension ($D_B$) and fractal span ($D_S$) of a lithotope boundary. $D_S = 2$ for a simple boundary and increases as the object becomes more complex. High $D_S$ is diagnostic of increasingly intricate shapes.

and decreasing abundance is approximately exponential (fig. 7). The implications of this observation are threefold: (1) the probability of encountering a highly intricate patch is low, (2) the relative proportions of simple- and complex-shaped patches is consistent between facies, and (3) the relationship between frequency of occurrence ($P$) and complexity of patchiness ($D_S$) is predictable according to equation [2]:

$$\log(P) = \log(D_S)^{-\alpha} + \beta,$$

where $\alpha$ is the decay constant and $\beta$ is the Y-intercept.

Reef areas dominated by a high coverage of coralline algae are conspicuous in attaining a considerably greater proportion of patches with very high boundary complexity, as compared with the other substrates (50% of coralline algae patches return $D_S > 5$ as opposed to <25%–35% for the remaining facies). The difference in complexity is likely related to the abundance of coralline algae in the fore-reef, high-energy zone, where accretion architecture is strongly governed by the topography and hydrodynamic regime of the reef crest. Though unique in its complex form, this facies conforms to the trajectory of the other facies in terms of the relationship between patch frequency and area (fig. 4).

Shape as a Function of Scale. By quantifying the shape of lithotope patches that comprise Pacific reefal landscapes, we can determine whether systematic trends in geometric form are present. Two descriptors modified from Peura and Iivarinen (1997) were used to characterize the shape of each of the 6439 facies polygons of the Pacific data set: (1) principal axes ratio (PAR), taken as the ratio of the longest segments of lines that cross each other orthogonally at the centroid of the patch, and (2) compactness, calculated as the ratio of the perimeter of a circle (with equal area to the patch) and the patch perimeter. Though considering shape through two different mechanisms, both metrics scale between 0 and 1, with high values indicative of circular shapes and low values returned for elongate forms.
From the preceding section, it is evident that complexity is independent of scale (fig. 6). We now investigate whether lithotope shape is also scale invariant [i.e., the landscape should possess all shape variants at all scales of consideration]. If this is the case, it follows that an image of self-similar facies patches would possess no cues as to its scale; an observer would be unable to estimate size on the basis of shape alone [e.g., fig. 8].

The smallest depositional province considered is Swains Atoll (American Samoa), with an area of 2 km² [the largest province is Hawaii, with 750 km²]. We take half of this least value [i.e., 1 km²] to represent a reasonable threshold below which lithotope patches are sufficiently small as to be unaffected by the size constraints imposed by shelf size (fig. 9, vertical dashed line). Patches <0.2 km² achieve the full spectrum of possible shape types with near-equal probability; that is, unit shape is independent of scale. For patches larger than 0.2 km² but smaller than the 1-km² threshold, there is a moderate abundance of elongate shapes. A possible explanation is that lithotope shape is frequently modulated by directional environmental forces at this scale [e.g., currents and wave action].

In the case of the fringing reefs considered in this study, the shelf is relatively narrow, exposed to waves and currents on the ocean side and protected by land on the other. Large lithotope units (defined as >1 km²) therefore tend to be narrow and elongate, as they reflect the shape constraints imposed by the host shelf [PAR and compactness approach 0; fig. 9]. For narrow depositional systems that cannot be viewed in their entirety [e.g., in the subsurface], it is conceivable that the relative proportion of rounded to elongate lithotopes may be used as a guide with which to estimate the total spatial extent of the hidden system. With the exception of units deposited under a directional regime at the small scale, the occurrence of only elongate lithotopes would indicate that the areas of the patches are starting to approach the scale of the host system. A diversity of patch shapes would suggest that the observed units are part of a system that is at least an order of magnitude larger than the largest rounded unit that can be resolved.

Self-similarity is a property of scale-invariant systems. Strictly, self-similar fractals are, by definition, isotropic [e.g., Turcotte 1997], and so the width [w] of a patch should approximately equal its length [l]. Fractals can also consist of anisotropic units [w ≠ l] and are termed “self-affine” (Dubuc et al. 1989). The shape metrics considered in figure 9 show a prevalence of elongate patches, a trend that strengthens with increasing area. What is therefore observed is a transition from only self-affine large lithotope units to a system at the small scale codominated by self-affine and self-similar patches. Despite this shift in affinity, the lithotope
patches follow a fractal frequency-area distribution when considered collectively [fig. 4].

Shape as a Function of Complexity. Patch shape is a critical variable controlling ecosystem function. A circular patch has a larger core than an elongated patch of the same area and is expected to contain a different suite of species, depending on their preference for edge and core habitat [e.g., Wagner and Edwards 2001]. Expansive core habitats containing “protruding bodies” such as seagrasses, filiform algae, rubble, and coral frameworks serve to impede hydrodynamic flow through friction [e.g., Nelson 1996]. Decrease in flow rate mediates settlement cues for pelagic propagules and promotes sedimentation [Abelson et al. 1994; Abelson and Denny 1997]. The abundance of core habitat is therefore important for the persistence and integrity of certain lithotopes. As such, in a reef setting, both the available space into which a patch mosaic must fit itself and the species assemblages characterizing each unit of that mosaic serve to influence the architecture of the resulting landscape. Patch shape and patch boundary complexity are therefore mutually connected. Our data confirm this hypothesis, with patch shape showing a strong correlation with boundary complexity $D_B$. Intricate patches ($D_B$ approaches 2) are commonly more rounded in their geometric form (principal axes lengths are similar), while more elongate patches display lower complexity [fig. 10].

The pattern of rounded shapes with more complex boundaries and less complex elongate units can likely be related to the role of depositional setting on the shape toward which a facies patch can evolve. Taking the case of lithotopes in the Saipan lagoon, as the reef crest is approached, we see that deposition is space limited by the proximity of deep water, resulting in preferential formation of elongate facies units [fig. 11, primary $Y$-axis]. While promoting elongation, the limited depositional space also promotes the deposition of lithotopes with lower boundary complexity as quantified using the box-counted fractal dimension [fig. 11, secondary $Y$-axis]. Conversely, in the back-reef lagoonal environment, accommodation space of a particular facies is less constrained, allowing the deposition of more isotropic patches. In this zone, hydrodynamic modulation is less pronounced than at the reef crest, and the shape of a patch perimeter is governed primarily by competitive biotic interactions with neighboring facies zones. Such competition operates at many spatial scales, promoting a tortuous boundary and therefore higher patch boundary complexity.

Why Are Modern Coral Reef Landscapes Fractal? Our analyses show that [1] the frequency-area relation of lithotope patches adheres to a power law and is therefore fractal, and [2] the majority of patches in the landscape have geometrically simple boundaries, although highly intricate units are present. Some of these objects are sufficiently intricate to be termed fractals; many are not. It can thus be concluded that the carbonate facies considered follow a fractal size distribution of objects of differing complexity. [3] With the exception of very large lithotope patches that are affected by the
dimension of their host shelf, all permutations of shape and boundary complexity are encountered at all spatial scales. The landscapes are scale invariant—a necessary condition for a fractal.

Over the four reefal provinces considered, numerous aspects of landscape patchiness are fractal. This conclusion is in good agreement with fractal scaling recognized in other carbonate depositional systems of more limited aerial extent (Rankey 2002; Schlager 2004, 2005), including modern reefal landscapes (Wilkinson and Drummond 2004; Purkis et al. 2006). In particular, the results agree with those of Purkis et al. (2005), who showed fractal behavior to exist in a mosaic of lithotopes atop a subtidal carbonate ramp in the Arabian Gulf. For the Gulf study, it was postulated that such patterning may be a direct result of cyclic disturbance regimes promoting fragmentation (Purkis and Riegl 2005).

This study is unique in providing an overview of system architecture at the basin scale, thereby exploring the upper limits of scaling laws in depositional settings. Coral reefs have both a strong sedimentary and a strong ecological component. Reef growth is largely a sedimentary process and can be modeled as such (Bosence et al. 1994; Warrlich et al. 2002), but in modern reefscapes, it is largely controlled by the fine-scale ecological processes of dispersal and settlement of frame-building corals and, to some degree, microbial action (Camoin et al. 2006). Power law scaling in purely sedimentary systems is just now becoming apparent (e.g., Carlson and Grotzinger 2001; Hergarten 2003), but the mechanisms of this scaling are as yet largely uninvestigated. In the simplest case of clastic deposits originating from a single point source and mediated only by hydrodynamic flow, the resulting sedimentary body shape is deltaic regardless of scale and expands downflow (e.g., Van Wagoner et al. 2003). This simple model of sedimentation may realistically account for self-similarity in fan-shaped sand bodies but does little to explain the fractal distribution of lithotopes on clastic shelves or shorelines (e.g., Rankey 2002). Molded by multiscale complex processes and lacking a single sedimentary point source, the architecture of carbonate landscapes is even less predictable than siliciclastic examples. This is because first, in the short term, carbonate deposition in space has a strong biotic component, and second, over geologic timescales, chemical effects play a major role in shaping deposition architecture. As such, there seems to be no single reason why the landscapes considered in this study have fractal attributes. To explore plausible explanations for the observed patterns, we therefore consider numerous aspects of the systems that may be relevant to the behavior.

We tackle the question in four sections. The first two consider the presence of chaos in the ecological functioning of reefal systems and the special influence of disturbance. The processes discussed typically act on spatial scales of millimeters to hun-
Figure 10. Patch shape [principal axes ratio; X-axis] versus box-counted fractal dimension of patch boundaries [Y-axis] for the four Pacific reef provinces. Note relation between increasing geometric complexity of lithotope boundaries and increasing patch roundness ($r^2 = 0.45$).

hundreds of kilometers—over time frames ranging from minutes to lifetimes. The latter two sections explore how a fractal may be promoted by mechanisms constructing reef habitats and the possible role of antecedent topography in the development of a fractal landscape. Here we shift our view of reefs to include processes over a longer temporal scale [i.e., thousands to hundreds of millions of years].

An Ecological Fractal. Carbonate depositional systems are built primarily by biological agents, and the behavior and life cycles of these organisms shape the structures that they create. Multiscale randomness [combinations of random processes operating at different resolutions] generate statistical fractal output patterns [De Cola 1989; Halley 1996; Halley and Kunin 1999] and are termed chaotic. Ecological modeling shows that chaos is not biologically unrealistic [O'Neill et al. 1982; Milton and Belair 1990; Pascual et al. 2002], and chaotic models are routinely used to represent ecological systems [Bascompte and Solé 1995; Seuront and Spilmont 2002; Rietkerk et al. 2004]. Randomness across temporal and spatial scale is a property of coral ecosystems, particularly to the stochastic settlement of coral recruits that dictate framework growth.

Chaotic ecosystems typically exhibit low minimum population sizes [Heino et al. 1997] and large fluctuations in population size through time. Coral prevalence within a benthic assemblage can fluctuate from >90% to <1% in a matter of months (Hughes 1989), returning to dominance in less than a decade if environmental conditions are favorable [Purkis and Riegl 2005]. Frame-building corals [the dominant architects of reefal systems] therefore display a population dynamic that can be considered chaotic. Consequently, coral populations, local biodiversity, and accretionary potential can exhibit extreme spatial variability through biotic action alone. In support, theoretical studies [Deutschman et al. 1993; Bascompte and Solé 1995] confirm that solely biotic interactions among individuals and populations produce space-time complexity in homogeneous environments. Complex patch mosaics resulting from biotic spatial processes [e.g., competitive, dispersal, and settlement dynamics] have also been shown to obey power laws [Li 2000], including power functions linking the decline in frequency with increasing patch area, as shown in this study (Urban 2005). The ecology of the studied depositional environments thus seems relevant to explaining the existence of a fractal.

The Role of Disturbance. Large expanses of a single facies unit in a landscape inhibit fractal scaling. When single large units are split into multiple small ones, any mechanism that serves to punctuate homogeneous units with dissimilar lithotopes greatly increases landscape patchiness. Fragmentation thus serves to increase the likelihood that patch frequency–area relations scale in log-log space, just as the number of small units increases with a corresponding decrease of large units. The distribution of “many small/few big patches” required for proportional scaling is promoted. As recognized by Done [1999], the frequency of disturbance events imposed on natural ecological spatial landscapes may also strongly alter the spatial configuration of a system. In areas where disturbance manifests as a reduction in the return interval between highly destructive events [hurricanes, temperature anomalies, outbreaks of coral predators], one might predict that the stage, age, and size-frequency distributions of coral populations and communities across regional seascapes will be skewed to those of earlier successional stages, with the relevance being that disturbance can preclude a particular assemblage from attaining spatial dominance, maintaining diversity in the system [e.g., Hughes 1989]. Ecological disruptions can thus promote small-scale patchiness and heterogeneity, necessary conditions for the maintenance of a complex lithotope mosaic that may already be, or have
Figure 11. Plot of distance from each patch centroid to the local reef crest versus patch shape [principal axes ratio \([\text{PAR}]\)] and patch boundary complexity [fractal dimension] for the 315 lithotopes comprising the Saipan lagoon. In close proximity to the reef crest, there is a tendency for the formation of elongate units with simple boundaries. Regression line between distance and \(\text{PAR}\) is solid; dashed line is regression between distance and box-counted fractal dimension \([D_B]\).
the potential to evolve to be, fractal. In the preceding section, it was recognized that the ecology of a depositional system is relevant to fractal patterning. By considering the possible impact of disturbance, we further hypothesize that perturbations to the ecological equilibrium also would promote a fractal landscape.

**Mechanisms of Construction.** Functioning in unison within a dynamic three-dimensional environment, reefscape development is driven by both sediment deposition and biotic action. This dual mechanism differs from the typically dominant role of sedimentation that drives the construction of the majority of Earth’s landscapes. The complex spatial distribution of reef lithotopes is thus governed by an interplay between the two modes of development. Of relevance is that an organism may respond to a landscape characteristic in a nonlinear way, such as requiring a specific minimum patch size or displaying threshold behavior in dispersal (Li 2000; Wagner and Fortin 2005). The cumulative effect of these processes is the formation of bistable systems (Rietkerk et al. 2004) or fractal patterns within complex thresholds (Pascual et al. 2002). The existing morphological complexity (abiotic or biotic in nature) further interacts with biotic processes to accentuate spatial patterns. In practical terms, the action of biotic and abiotic effects may be inseparable, and little practical value is attained by attempting to partition these factors. As discussed by Purkis et al. (2005) for the Arabian Gulf, abiotic seafloor lithification greatly reduces the rate of mechanical erosion, making sand sheets more resistant to lateral displacement. A similar stabilization is observed in the short term through the colonization of soft sediment by algal or seagrass stands. In both cases, patchiness is promoted, and a complex landscape mosaic results. It is evident that the abiotic-biotic interplay dictating reef structure may serve to promote spatial complexity.

**Antecedent Topography.** The topographic alteration of a carbonate platform through karstification during subaerial exposure can be dramatic. Even if dissolution is minor, the altered topographic highs can become sites of accelerated accretion, thus accentuating subsequent depositional morphology (e.g., Esker et al. 1998). Repeated regression/transgression may therefore further amplify topographic complexity, which translates to increased heterogeneity in plan view as the biotic landscape architects order themselves among their preferred depth regimes. In this respect, the role of antecedent topography cannot be underestimated as a template for the modern expression of landscape heterogeneity. This promotion of complexity through karst action is, however, not guaranteed. Cycles of regression, exposure, and transgression can also serve to diminish the persistence of antecedent pattern by eroding topographic variability. Unlike the case of ecological interaction on short timescales, the influence of past sea level over geologic time frames can both promote and retard spatial complexity in carbonate depositional environments.

In summary, the above sections demonstrate that multiple factors acting alone or in concert may explain the existence of fractal behavior. Ecological interactions can result in power function distributions of patch area–frequency relations. Ecological disturbance likely aids this relation by segmenting homogeneous areas and promoting a landscape consisting of a greater number of lithotope types. By promoting heterogeneity, disturbance may also be an important force acting to increase patch boundary complexity in depositional environments that are not space limited (e.g., by proximity to deep water). Even in nondisturbed landscapes, the interplay between abiotic and biotic deposition appears to be a force that enhances spatial complexity, and over geologic timescales, the influence of changing sea level may serve to further increase spatial heterogeneity. Regardless of the driving mechanism that results in a class of lithotope becoming sufficiently complex to be fractal (for frequency-area relations and/or shape complexity), if sufficiently prevalent, it may start to alter the structure of the landscape as a whole. Once a dominant landscape architect adopts a fractal distribution, for example, other facies will follow by virtue of the fact that they fill the “negative” space in the landscape. An example is the highly patchy nature of coral settlement and accretion, forming a template to which mobile sand sheets will adhere. A fractal pattern in this case may simply be a reflection of another underlying fractal (e.g., Halley et al. 2004). This multiplicity in scale may lead to the notion that carbonate depositional landscapes can be considered superimposed fractal mosaics.

**Conclusions**

This study is successful in depicting that the statistical pattern of size, shape, and complexity of patchiness in a suite of modern carbonate depositional settings is predictable by virtue of the existence of fractal scaling. The consequence is that (1)
the complex heterogeneity of carbonate reservoirs can be described by a few pertinent indices and (2) the statistical distribution of patchiness is predictable at scales beyond that at which the system is observed. The true persistence of scaling predictability requires further study. However, our work in 26 Pacific sites shows robust scaling spanning $10^3$–$10^8$ m$^2$ (five orders) for eight facies types. With the dominant facies in these depositional systems all adhering to independent scaling laws, it is indicated that modern coral reef landscapes (and therefore subsurface carbonate reservoirs) can be considered scale-invariant lithotope mosaics with a fractal size distribution. The span of fractal behavior is sufficient to be of significant utility in the prediction of the architecture of both modern and ancient carbonate depositional systems.

ACKNOWLEDGMENTS

We wish to thank A. Shapiro and M. Anderson for aid in interpreting National Oceanic and Atmospheric Administration (NOAA) map products. This publication is a result of funding from the NOAA Center for Sponsored Coastal Ocean Science, under award NA04NOS4260065 to Nova Southeastern University for the National Coral Reef Institute (NCRI). Invaluable comments, for which we are extremely grateful, were provided by W. Schlager and two anonymous reviewers. This is NCRI contribution 84.

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